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Quasi-Static Thermoelastic Problem with Moving Heat Source in One Directional Rod with Robin's Boundary Conditions

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Abstract:

We have considered one-dimensional rod of length a *occupying the region* $0 \le x \le a$. *Initial temperature of the rod is zero placed in an ambient temperature zero. The rod is subjected to the activity of instantaneous moving point heat source at the point* x' *which changes its place along* x *- axis, moving with constant velocity* u. *The temperature sharing of the rod in one dimension is described by the differential equation of heat conduction with heat generation terms. Solution of the temperature distribution is obtained by solving heat conduction equation and relative thermal stresses are obtained.*

1. Introduction

In 2014 D T Solanke and M H Durg have studied temperature distribution and thermal stresses in thin solid cylinder, thin hollow cylinder, thin rectangular plate, thick circular plate, one dimensional rod in [9] to [25]. Now in this paper authors determined the temperature distribution and thermal stresses in one-dimensional rod with moving point heat source with quasi-stationary condition. This is new contribution in the field of thermo elasticity.

2. Formulation of the problem for Robin's boundary condition

The temperature sharing of the rod in one dimension is described by the differential equation of heat conduction with heat generation term, as in [7] is given by

by

$$\frac{\partial^2 T}{\partial x^2} + \frac{1}{k} g = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.1)$$

where $T = T(x,t)$ is temperature distribution, k is thermal conductivity of the material of the rod, $\alpha = \frac{k}{c_p \rho}$ is thermal diffusivity, ρ is density, c_p is specific heat of the material of the rod and g is heat generation term. Now consider an instantaneous moving point heat source located at point x' and releasing its heat spontaneously at time t. Such point moving heat source in one dimension rod is given by the delta function

$$g(x,t) = g_i^p \delta(x - x'), \qquad (2.2)$$

where $x' = ut$

Hence (2.1) reduces to

$$\frac{\partial T}{\partial x^2} + \frac{1}{k} g_i^p \delta(x - ut) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.4)$$

3.Robin's boundary condition and initial condition

We formulate the homogeneous Dirichlet's boundary conditions and initial condit

$$k\frac{\partial T}{\partial x} - hT = 0 \quad \text{at} \quad x = 0, \quad t = 0 \qquad (3.1)$$

$$k\frac{\partial T}{\partial x} + hT = 0 \quad \text{at} \quad x = a, \quad t = 0 \qquad (3.2)$$

where k is thermal conductivity and h is heat transfer coefficient of the material of the rod.

In the solution of the moving heat source problem it is convenient to let the coordinate system move with the source. This is achieved by introducing a new coordinate X defined by

$$X = x - ut \qquad (3.3)$$

Hence (2.4) reduces to

$$\frac{\partial^2 T}{\partial X^2} + \frac{1}{k} g_i^p \delta(X) = \frac{1}{\alpha}\left[\frac{\partial T}{\partial t} - u\frac{\partial T}{\partial X}\right] \qquad (3.4)$$

4. Quasi stationary condition

Stationary medium means initially at zero temperature. Hence quasi-stationary condition is mathematically defined by setting $\frac{\partial T}{\partial t} = 0$ and hence (3.4) reduces to

$$\frac{\partial^2 T}{\partial X^2} + \frac{u}{\alpha}\frac{\partial T}{\partial X} = -\frac{1}{k} g_i^p \delta(X) \qquad (4.1)$$

above equation can be transformed into a more convenient form by introducing a new variable $\theta(X)$ defined by

$$T(X) = \theta(X) e^{-\frac{u}{2\alpha}X} \qquad (4.2)$$

Hence (4.1) reduces to

$$\frac{\partial^2 \theta}{\partial X^2} - \left(\frac{u}{2\alpha}\right)^2 \theta = -\frac{1}{k} g_i^p \delta(X) e^{\frac{u}{2\alpha}X} \qquad (4.3)$$

5. Solution of temperature distribution

Solution of (4.3) is

$$\theta = c_1 e^{\left(\frac{u}{2\alpha}X\right)} e^{\left(-\frac{u^2}{2\alpha}t\right)} + c_2 e^{\left(-\frac{u}{2\alpha}X\right)} e^{\left(\frac{u^2}{2\alpha}t\right)} + \frac{\alpha g_i^p}{ku}\left[e^{\left(-\frac{u}{2\alpha}X\right)} e^{\left(\frac{3u^2}{2\alpha}t\right)} - e^{\left(\frac{u}{2\alpha}X\right)} e^{\left(-\frac{u^2}{2\alpha}t\right)}\right] \qquad (5.1)$$

From (4.2) and (5.1), we obtain

$$T(x,t) = c_1 + c_2 e^{\left(-\frac{u}{\alpha}x\right)} e^{\left(\frac{u^2}{\alpha}t\right)} + \frac{\alpha g_i^p}{ku}\left[e^{\left(-\frac{u}{\alpha}x\right)} e^{\left(\frac{2u^2}{\alpha}t\right)} - 1\right] \qquad (5.2)$$

Applying the condition (3.1) and (3.2), we get

$$c_1 = \frac{g_i^p}{k}\left[\frac{\frac{\lambda_0 \alpha}{u}\left(e^{-\frac{u}{\alpha}a} - 1\right) - \left(e^{-\frac{u}{\alpha}a} + 1\right)}{\lambda_0\left(e^{-\frac{u}{\alpha}a} - 1\right) - \frac{u}{\alpha}\left(e^{-\frac{u}{\alpha}a} + 1\right)}\right]$$

$$c_2 = \frac{g_i^p}{k}\left[\frac{\left(e^{-\frac{u}{\alpha}a} + 1\right) - \frac{\lambda_0 \alpha}{u}\left(e^{-\frac{u}{\alpha}a} - 1\right)}{\lambda_0\left(e^{-\frac{u}{\alpha}a} - 1\right) - \frac{u}{\alpha}\left(e^{-\frac{u}{\alpha}a} + 1\right)}\right] \qquad (5.4)$$

where $\lambda_0 = \frac{h}{k}$

From (5.2), (5.3) and (5.4), we get

$$T(x,t) = \frac{g_i^p}{k}\left[\frac{\frac{\lambda_0 \alpha}{u}\left(e^{-\frac{u}{\alpha}a} - 1\right) - \left(e^{-\frac{u}{\alpha}a} + 1\right)}{\lambda_0\left(e^{-\frac{u}{\alpha}a} - 1\right) - \frac{u}{\alpha}\left(e^{-\frac{u}{\alpha}a} + 1\right)}\right]\left[1 - e^{\left(-\frac{u}{\alpha}x\right)} e^{\left(\frac{u^2}{\alpha}t\right)}\right] - \frac{\alpha g_i^p}{ku}\left[1 - e^{\left(-\frac{u}{\alpha}x\right)} e^{\left(\frac{2u^2}{\alpha}t\right)}\right] \qquad (5.5)$$

6. Thermoelastic problem

Let us introduce a thermal stress function χ related to component of stress in the material of one dimensional rod as in [8]

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \qquad (6.1)$$

with the boundary condition

$$\sigma_{yy} = 0 \text{ at } x = 0,\ x = a, \qquad (6.2)$$

where

$$\chi = \chi_c + \chi_p \qquad (6.3)$$

χ_c is complementary function and χ_p is particular function. χ_c and χ_p are governed by a linear homogeneous differential equation and linear non-homogeneous differential equation.

$$\nabla^4 \chi_c = 0 \qquad (6.4)$$

$$\nabla^4 \chi_p = -\lambda E \nabla^2 \Gamma, \qquad (6.5)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}$ (6.7)

Γ is temperature change $\Gamma = T - T_i$, where T_i is initial temperature which is zero.

7 Solution of the thermoelastic problem

$$\Gamma = \frac{g_i^p}{k}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right]\left[1-e^{\left(-\frac{u}{\alpha}x\right)}e^{\left(\frac{u^2}{\alpha}t\right)}\right] - \frac{\alpha g_i^p}{ku}\left[1-e^{\left(-\frac{u}{\alpha}x\right)}e^{\left(\frac{2u^2}{\alpha}t\right)}\right] \quad (7.1)$$

Let the compatible form of χ_c satisfying (6.4) be

$$\chi_c = cx^3 + dx^2 \quad (7.2)$$

Let compatible form of χ_p satisfying (6.5) be

$$\chi_p = -\lambda E\frac{g_i^p}{k}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right]\left[\frac{x^2}{2}-\frac{\alpha^2}{u^2}e^{-\frac{u}{\alpha}x}e^{\frac{u^2}{\alpha}t}\right] - \frac{\alpha g_i^p}{ku}\left[\frac{x^2}{2}-\frac{\alpha^2}{u^2}e^{-\frac{u}{\alpha}x}e^{\frac{2u^2}{\alpha}t}\right] \quad (7.3)$$

$\chi = \chi_c + \chi_p$ gives

$$\chi = cx^3 + dx^2 - \lambda E\frac{g_i^p}{k}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right] \times\left[\frac{x^2}{2}-\frac{\alpha^2}{u^2}e^{-\frac{u}{\alpha}x}e^{\frac{u^2}{\alpha}t}\right] - \frac{\alpha g_i^p}{ku}\left[\frac{x^2}{2}-\frac{\alpha^2}{u^2}e^{-\frac{u}{\alpha}x}e^{\frac{2u^2}{\alpha}t}\right] \quad (7.4)$$

From (6.1) and (7.4), we obtain

$$\sigma_{yy} = 6cx + 2d - \lambda E\frac{g_i^p}{k}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right] \times\left[1-e^{-\frac{u}{\alpha}x}e^{\frac{2u^2}{\alpha}t}\right] - \frac{\alpha g_i^p}{ku}\left[1-e^{-\frac{u}{\alpha}x}e^{\frac{2u^2}{\alpha}t}\right] \quad (7.5)$$

Applying the condition (6.2), we get

$$c = \frac{g_i^p}{6ak}\left[1-e^{-\frac{u}{\alpha}a}\right]\left\{\lambda E e^{\frac{u^2}{\alpha}t}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right] + \frac{\alpha}{u}e^{\frac{2u^2}{\alpha}t}\right\} \quad (7.6)$$

$$d = \frac{g_i^p}{k}\left\{\frac{\lambda E}{2}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right]\left[1-e^{\frac{u^2}{\alpha}t}\right] + \frac{\alpha}{2u}\left[1-e^{\frac{2u^2}{\alpha}t}\right]\right\} \quad (7.7)$$

Inserting the value of c and d in (7.5), we get

$$\sigma_{yy} = \frac{g_i^p}{ak}x\left[1-e^{-\frac{u}{\alpha}a}\right]\left\{\lambda E e^{\frac{u^2}{\alpha}t}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right] + \frac{\alpha}{u}e^{\frac{2u^2}{\alpha}t}\right\} + \frac{g_i^p}{k}\left\{\lambda E\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right] \times\left[1-e^{\frac{u^2}{\alpha}t}\right] + \frac{\alpha}{u}\left[1-e^{\frac{2u^2}{\alpha}t}\right]\right\}$$

$$-\lambda E\frac{g_i^p}{k}\left[\frac{\frac{\lambda_0\alpha}{u}(e^{-\frac{u}{\alpha}a}-1)-(e^{-\frac{u}{\alpha}a}+1)}{\lambda_0(e^{-\frac{u}{\alpha}a}-1)-\frac{u}{\alpha}(e^{-\frac{u}{\alpha}a}+1)}\right]\left[1-e^{-\frac{u}{\alpha}x}e^{\frac{2u^2}{\alpha}t}\right] \quad (7.8)$$

8. Conclusion

In this paper we have determined the time dependent temperature distribution and thermal stresses in one dimensional rod with moving point heat source in stationary condition with analytical approach. by giving particular values to the parameters one can obtain their desired results. From the stress equation we observe that initially ($t=0$) stress vanishes.

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